A Parameterization for the Effects of Ozone on the Wave Driving Exerted by Equatorial Waves in the Stratosphere

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ABSTRACT

An equatorial $\beta$-plane model of the tropical stratosphere is used to examine the effects of ozone on Kelvin, Rossby–gravity, equatorial Rossby, inertia–gravity, and smaller-scale gravity waves. The model is composed of coupled equations for wind, temperature, and ozone volume mixing ratio, which are linearized about a zonally averaged background state. Using the Wentzel–Kramers–Brillouin (WKB) formalism, equations are obtained for the vertical spatial scale, spatial damping rate, and amplitude of the waves. These equations yield an analytical expression for the ozone-modified wave driving of the zonal-mean circulation. The expression for the wave driving provides an efficient parameterization that can be implemented into models that are unable to spontaneously generate the ozone-modified, convectively coupled waves that drive the quasi-biennial and semiannual oscillations of the tropical stratosphere.

The effects of ozone on the wave driving, which are strongly modulated by the Doppler-shifted frequency, are maximized in the upper stratosphere, where ozone photochemistry and vertical ozone advection combine to augment Newtonian cooling. The ozone causes a contraction in spatial scale and an increase in the spatial damping rate. In the midstratosphere to lower mesosphere, the ozone-induced increase in wave driving is about 10%–30% for all wave types, but it can be as large as about 80% over narrow altitude regions and for specific wave types. In the dynamically controlled lower stratosphere, vertical ozone advection dominates over meridional ozone advection and opposes Newtonian cooling, causing, on average, a 10%–15% reduction in the damping rate.

1. Introduction

The quasi-biennial oscillation (QBO) in zonal wind in the tropical lower stratosphere is among the most robust wave-driven circulations in Earth’s atmosphere (Baldwin et al. 2001). Indeed, the QBO signal extends far beyond its tropical seat of origin to affect, for example, the northern annular mode (Coughlin and Tung 2001), the Northern Hemisphere polar vortex (Lu et al. 2008), and the timing of stratospheric sudden warmings (Gray et al. 2004). Yet despite the robustness of the QBO, it is among the most difficult atmospheric circulations to spontaneously generate and accurately represent in general circulation models (GCMs).

The difficulty in generating a realistic QBO hinges largely on two factors. First, the models must accurately spawn the broad spectrum of convectively coupled, equatorially trapped waves that drive the QBO. These waves include Kelvin, Rossby–gravity, equatorial Rossby, inertia–gravity, and smaller-scale gravity waves. Second, the models must accurately represent the mechanical and radiative-photochemical damping mechanisms...
that operate on the waves in the stratosphere. Meeting these two model requirements would produce, in principle, the necessary momentum fluxes needed to generate a QBO consistent with observations.

The above-mentioned modeling challenges notwithstanding, studies have made progress in spontaneously generating, without gravity wave parameterizations, “QBO-like” circulations in GCMs (Takahashi 1999; Hamilton et al. 2001; Watanabe et al. 2008; Kawatani et al. 2009; Kawatani et al. 2010). Perhaps the most successful spontaneously generated QBO simulation is that of Kawatani et al. (2010), who used an atmospheric GCM with horizontal and vertical resolutions of 60 km and 300 m, respectively. These spatial resolutions produced a broad spectrum of convectively coupled waves spanning zonal wavenumbers $s = 1–213$ (waves $s > 100$ were shown to be crucial to the westerly shear phase of the QBO). Based on a 3-yr model simulation, the amplitude and structure of the modeled QBO was in reasonable agreement with observations. The average period of the modeled QBO, however, differed sharply from the observed QBO: about 15 months (model) versus about 27 months (observations). Kawatani et al. attributed the difference between the modeled and observed QBO periods to the absence of interactive ozone in their model, which Shibata and Deushi (2005) have shown can increase the period of the QBO by a factor of about 1.8.

The importance of ozone to the QBO, as well as to the semiannual oscillation (SAO) of zonal-mean wind in the tropical upper stratosphere and lower mesosphere, is well established (Leovy 1966; Echols and Nathan 1996; Cordero and Nathan 2000; Shibata and Deushi 2005). Yet, our understanding of how the distribution, transport, and chemistry of ozone combine to affect the QBO and SAO remains incomplete. Moreover, of the relatively few models that can spontaneously generate a QBO without gravity wave parameterizations, even fewer include three-dimensional, interactive ozone.

There are two central challenges associated with ozone and its effects on the QBO. The first challenge hinges on understanding the causal relationships between the wave and zonal-mean fields, and the interactions involving ozone transport, ozone photochemistry, and radiative transfer. The second challenge hinges on balancing the need for sufficient model resolution to spontaneously generate and accurately reproduce a QBO, with the need to reduce computational time in order to obtain reliable statistics, especially in climate studies where multidecadal simulations may be required. Although the ability to combine long-time integrations with high spatial resolution continues to improve with increasing computer resources, challenges remain. For this reason, mechanistic models continue as important tools for easing exposure of causal relationships, for guiding experiments, and for interpreting results obtained from high-resolution, coupled chemistry models of the tropical stratosphere.

Mechanistic models have indeed advanced our understanding of how ozone transport and ozone photochemistry affect the zonal-mean circulation of the tropical stratosphere (Echols and Nathan 1996; Cordero et al. 1998; Cordero and Nathan 2000, 2005). Cordero et al., for example, used a one-dimensional (in height) model to examine how the ozone-modified Kelvin and Rossby–gravity waves together affected the model QBO. They found that the ozone increased the magnitude of the zonal-mean wind by about 1–2 m s$^{-1}$ and increased the period of the QBO by about 2 months. The ozone-induced increase in the period of the QBO is consistent with Shibata and Deushi (2005), although the increase in period is considerably longer (~21 months) in Shibata and Deushi. These findings, however, differ from those of Cordero and Nathan (2000), who used a 2.5-dimensional model of the equatorial stratosphere that accounted for zonal-mean– and wave–ozone feedbacks involving Kelvin and Rossby–gravity waves. Cordero and Nathan found that the ozone feedbacks had only a small effect on the period of the QBO, though the zonal wind and temperature QBOs were 10%–20% larger than the simulations without the ozone feedbacks.

Because the models are formulated differently, it is difficult to determine if the agreement between the Cordero et al. (1998) and Shibata and Deushi (2005) findings is fortuitous, or if the difference between the Cordero and Nathan (2000) and Shibata and Deushi findings is an artifact of the model differences. For example, Cordero et al. used a one-dimensional model with only two waves, with each wave coupled to ozone, whereas Shibata and Deushi used a three-dimensional, coupled chemistry model with resolved zonal waves $s = 1–42$, with the smaller-scale gravity waves parameterized following Hines (1997). The gravity wave parameterization, however, did not include feedbacks with the wave–ozone field. How Shibata and Deushi’s results might change with an ozone-modified gravity wave parameterization is unclear. Cordero and Nathan’s (2000) model, however, only accounted for Kelvin and Rossby–gravity waves, though the forcing amplitudes at the lower boundary were chosen to produce a net momentum driving consistent with observations. How Cordero and Nathan’s results might change with the inclusion of an ozone-modified, high-frequency gravity spectrum also is unclear. Despite the various model limitations, there is agreement among the models and other studies regarding two features. First, models that seek to produce a realistic QBO must have a broad wave spectrum that
includes high-frequency gravity waves (Dunkerton 1997). Second, wave–ozone feedbacks make an important contribution to the wave damping, which affects the zonal-mean wave fluxes and thus the driving of the zonal-mean circulation (Cordero et al. 1998).

Motivated largely by the studies of Dunkerton (1997) and Cordero et al. (1998), we combine and extend their studies to examine the effects of ozone on a broad spectrum of tropical waves. In so doing, we analytically derive expressions for the spatial scale, spatial damping rate, and amplitude of the waves, which include the effects of ozone transport and ozone photochemistry. These expressions clearly illuminate the physics of the coupled wave–ozone feedbacks. Moreover, we show that the equations for the vertical spatial scale, spatial damping rate, and amplitude combine to yield an expression for the ozone-modified wave driving on the zonal-mean flow. The ozone-modified wave driving provides an efficient parameterization that can be implemented into models that are unable to spontaneously generate the ozone-modified, convectively coupled waves that drive the quasi-biennial and semiannual oscillations of the tropical stratosphere.

2. The model

We use a model atmosphere centered on an equatorial β plane (Cordero et al. 1998). The model consists of wave equations that describe zonal and meridional wind, mass, temperature, ozone volume mixing ratio, and hydrostatic balance. We write these equations in log-pressure coordinates and linearize about a vertically sheared, basic-state zonal flow that is in radiative–photochemical equilibrium:

\[
\begin{align*}
\frac{\partial}{\partial t} + \bar{u} \frac{\partial}{\partial x} + \bar{v} \frac{\partial}{\partial y} + \frac{\partial}{\partial z} & = 0, \\
\frac{\partial}{\partial t} + \bar{u} \frac{\partial}{\partial x} & = 0, \\
\frac{\partial}{\partial t} + \bar{u} \frac{\partial}{\partial x} & = 0, \\
\frac{\partial}{\partial t} + \bar{u} \frac{\partial}{\partial x} & = 0, \\
\frac{\partial}{\partial t} + \bar{u} \frac{\partial}{\partial x} & = 0, \\
\frac{\partial}{\partial t} + \bar{u} \frac{\partial}{\partial x} & = 0.
\end{align*}
\]

We denote perturbation and zonal-mean quantities by primes and overbars, respectively. All of the variables in (1)–(6) are defined in Table 1.

Equations (1)–(6) form a closed set that describe the linear response of a tropical wave field to coupled interactions involving the dynamical circulation, longwave radiative transfer, ozone transport, and ozone photochemistry.\(^1\)

\(^1\) We use a linear representation of the ozone photochemistry, which, as Hitchcock et al. (2010) notes, compares well to the nonlinear representation of the ozone photochemistry.
The radiative–photochemical portion of the model is based on Nathan and Li (1991) and Nathan and Cordero (2007). The radiative–photochemical feedbacks appear on the right-hand side (rhs) of the thermodynamic energy equation (4) and ozone continuity equation (5). The first term on the rhs of (4) represents longwave radiational cooling, which we model as Newtonian cooling based on the scale-dependent parameterization of Fels (1982). The second term on the rhs of (4) represents the local ozone heating. The first two terms on the rhs of (5) represent the temperature-dependent ozone production and destruction rate, which includes heating and reaction rates for ozone and oxygen (Lindzen and Goody 1965), and catalytic ozone loss cycles involving odd chlorine, odd nitrogen, and odd hydrogen (Hartmann 1978). Following Strobel (1978), we have incorporated the effects of multiple scales. For the tropical waves considered here, however, the shielding effects are inconsequential to the vertical spatial scales and spatial damping rates. This can be shown by noting, as in Echols and Nathan (1996), that \( \gamma' \) is approximately oscillatory in the vertical, so that we can choose a harmonic solution for \( \gamma' \) such that the integrals in the shielding terms can be written as

\[
\chi' = \int z \frac{\rho(z')}{\rho_0} \gamma'(x, y, z', t) \, dz'
= \gamma' \rho \exp(-z/H) \left( 1 + \frac{im_0 H}{1 + m_0 H^2} \right),
\]

where \( m_0 \) is the (real) vertical wavenumber. We calculated the ratio of the ozone shielding effects to the local ozone effects in (4) and (5) and found the ratio is less than 1% over most of the altitude range. Thus, we neglect the shielding effects in (4) and (5).

Figure 1a shows the distribution of the Newtonian cooling coefficient \( \sigma_{NC} \) for a vertical wavelength of 9 km. For the numerical calculations we present later, however, we calculate \( \sigma_{NC} \) based on the local vertical wavelength for the wave considered. Figures 1b–d show the photochemical coefficients \( A, B, \) and \( C \), which we calculated for 15°N using a September climatological zonal-mean ozone profile based on McPeters et al. (1984) and Keating and Young (1985), and a zonal-mean temperature profile based on Fleming et al. (1988); 15°N is chosen because \( \Psi \) is largest at this latitude in the tropical stratosphere.

3. Spatial scale, spatial damping rate, and amplitude

To derive the local, ozone-modified spatial scale, spatial damping rate, and amplitude for each wave type, we assume that the background fields of wind, temperature, and ozone are slowly varying in the vertical. We also assume that the radiative–photochemical forcing terms are small. As in Cordero et al. (1998), these two assumptions are made explicit by introducing the slowly varying vertical coordinate \( \xi = \mu z, \) where \( \mu \ll 1 \), and scaling the radiative–photochemical forcing such that the right-hand side of (4) and (5) are \( O(\mu) \). With this rescaling, the spatial variations of the background fields are such that \( \pi(\xi), T(y, \xi), \) and \( \Psi(y, \xi) \). The wave fields in (1)–(6) are chosen to be Wentzel–Kramers–Brillouin (WKB) in form (Bender and Orszag 1978):

\[
\Re'(x, t; \xi) = \Re \left[ \Re(y, \xi) \exp \left( i(kx - \alpha t) + i \left[ \mu^{-1}m_0(\xi) + m_1(\xi) \right] \frac{z}{2H} \right) \right].
\]

In (8), \( x = (x, y, z) \) is the position vector; \( \Re' = \{u', v', w', T', \Phi', \gamma'\} \) represents the wave fields in wind, geopotential, and ozone, while the amplitude of the fields is denoted by \( \Re(y, \xi) ; \alpha \) is the ground-based frequency; and \( k = s(a, c) \cos \theta^{-1} \), where \( s \) is the quantized zonal wavenumber, \( a_c \) is the radius of Earth, and \( \theta \) is latitude. As in (7), \( m_0 \) is the local, real, vertical wavenumber, while \( m_1 = m_{1z} + im_{1i} \) is the local, ozone-modified, complex correction to \( m_0 \). More specifically, the real part of \( m_1 \) is an \( O(\mu) \) correction to the vertical spatial scale, which is proportional to \( m_{1z}^2 \); the imaginary part of \( m_1 \) controls the vertical spatial amplification or damping.

Insertion of (8) into (1)–(6) yields an asymptotically expanded system about orders of \( \mu \). The \( O(1) \) system, which is identical to Matsumo (1966), yields the dispersion relation and the meridional and vertical structures of the waves. The dispersion relation is

\[
\left( \frac{m_0^W \omega}{N} \right)^2 - k^2 = \frac{\beta k}{\omega} = (2n + 1) \frac{\beta m_0^W}{N}.
\]

The superscript \( W \) distinguishes the wave type: Kelvin wave (\( W = \text{KW} \)), Rossby–gravity wave (\( W = \text{RG} \)), equatorial Rossby wave (\( W = \text{ER} \)), inertia–gravity wave (\( W = \text{IG} \)), and high-frequency gravity wave (\( W = \text{GW} \)). In deriving the dispersion relation (9), we have neglected a term arising from \( \exp(z/2H) \) in (8) because it provides only a small linear correction (e.g., Andrews et al. 1987). In (9), \( \omega = \sigma - k\bar{u} \) is the Doppler-shifted
frequency, and \( n \) is a positive integer that corresponds to a meridional wave mode. In appendix A we provide the dispersion relation and the meridional and vertical structures for each wave type.

The radiative–photochemical forcing enters the system at \( O(\mu) \). Applying a solvability condition at this order yields the local vertical wavenumber \( m_{1,r} \), which is inversely proportional to the vertical scale, the local spatial damping rate \( m_{1,i} \), and the slowly varying amplitude \( R(y, z) \) as shown:

\[
m_{1,r}^W = \frac{m_0^W}{2\omega} \left( \frac{\omega}{B} \alpha_{\text{OP}}^W - \frac{B}{\omega} \alpha_{\text{VOA}}^W - \frac{\omega}{B} \alpha_{\text{MOA}}^W \right),
\]

\[
m_{1,i}^W = -\frac{m_0^W}{2\omega} (\alpha_{\text{NC}}^W + \alpha_{\text{OP}}^W - \alpha_{\text{VOA}}^W - \alpha_{\text{MOA}}^W),
\]

\[
R^W(y, \zeta) = a_0 |m_0^W|^{1/4} \exp[\Pi(y, \zeta; m_0^W)],
\]

where

\[
\alpha_{\text{OP}}^W = \frac{ABC}{(B^2 + \omega^2)}
\]

\[
\alpha_{\text{VOA}}^W = \frac{A\omega^2}{N^2(B^2 + \omega^2)} \left[ \int_{-\infty}^{\infty} \nabla_y(y, \zeta) D(y; m_0^W) \, dy \right]
\]

\[
\alpha_{\text{MOA}}^W = \frac{AB}{(N^3Bm_0^W)^{1/2}(B^2 + \omega^2)} \left[ \int_{-\infty}^{\infty} \nabla_y(y, \zeta) E(y; m_0^W) \, dy \right].
\]

In (10), \( m_{1,r} \) depends on three ozone effects: ozone photochemistry \( \alpha_{\text{OP}} \), vertical ozone advection \( \alpha_{\text{VOA}} \), and...
meridional ozone advection $a_{\text{MOA}}$. In (11), $m_{1,J}$ depends on the same three ozone effects as $m_{1,r}$, plus Newtonian cooling. In (12), $a_0$ is a constant, which can be determined by application of a lower boundary condition (e.g., vertical velocity). In (12), (14), and (15), $D(y;m_0^w)$, $D(y;m_0^R)$ and $E(y;m_0^w)$, which are defined in appendix B, are functions of the meridional and vertical structures for a given wave type.

The radiative–photochemical coefficients in (10) and (11) modify a wave’s vertical spatial scale and spatial damping rate, respectively. If $m_{1,J}$ opposes (augments) $m_0$, then the vertical scale expands (contracts); if $m_{1,J}$ is negative (positive), then the wave spatially amplifies (damps). Newtonian cooling always damps a wave, irrespective of wave type. Thus, ozone either augments or opposes the Newtonian cooling.

The central goal of our study is to determine how the effects of ozone on the equatorial waves in the tropical stratosphere drive the zonal-mean circulation. The ozone-modified wave driving on the zonal-mean circulation is measured by divergence of the Eliassen–Palm (EP) flux, which can be written as (Echols and Nathan 1996):

$$\mathbf{V} \cdot \mathbf{F} = \frac{\partial}{\partial y} \rho \left( \frac{R T}{H N^2} \mathbf{v} \mathbf{v}^T - \mathbf{u} \mathbf{v}^T \right) + \frac{\partial}{\partial \z} \rho \left( \frac{R T}{H N^2} \mathbf{v} \mathbf{v}^T - \mathbf{u} \mathbf{v}^T \right).$$

(16)

Because we have obtained expressions for the ozone-modified wave fields in wind and temperature in (8), which depend on the spatial scale in (10), spatial damping rate in (11), and wave amplitude in (12), the wave driving on the zonal-mean circulation in (16) can be calculated. All that is required is knowledge of the background distributions of wind, temperature, and ozone, and the characteristics of a specific wave type: zonal wave-number, meridional mode, and frequency. Thus the ozone-modified wave driving in (16) provides an efficient parameterization that can be implemented into models that are unable to spontaneously generate the ozone-modified, convectively coupled waves that drive the quasi-biennial and semiannual oscillations of the tropical stratosphere.

a. Basic states and radiative–photochemical coefficients

We calculate the spatial scale, spatial damping rate, and amplitude based on analytical expressions for the background distributions of wind, temperature, and ozone that are consistent with observations. For the basic-state wind, we use two profiles that are representative of the descending easterly and descending westerly phases of the QBO:

$$\pi(z) = \pm 20 \tanh[(z - z_m)/\beta],$$

(17)

where $z$ is the altitude above mean sea level and $z_m = 27$ km is the level of maximum wind shear (see Fig. 2a). We choose the following expression for the basic-state ozone distribution to ease the integrations in (14) and (15):

$$\tilde{\gamma}(y_L,z) = \frac{1}{2} y_L \tilde{\gamma}(y_{15},z)^2 + \tilde{\gamma}(y_{\text{EO}},z).$$

(18)

The corresponding meridional and vertical ozone gradients are

$$\tilde{\gamma}(y_L,z) = \frac{1}{2} y_L \tilde{\gamma}(y_{15},z)^2 + \tilde{\gamma}(y_{\text{EO}},z).$$

(19)

$$\tilde{\gamma}(y_L,z) = a y_L \tilde{\gamma}(y_{15},z)^2,$$

(20)

where $L$ denotes the latitude where (18)–(20) are evaluated, $y_L = a y_L$, and $a = 1.0 \times 10^{-8} \text{ m}^{-1}$ is the inverse distance from the equator ($y_{\text{EO}}$) to $15^\circ$N ($y_{15}$). The terms with a caret on the rhs of (18)–(20) represent the vertical distributions of zonal-mean ozone obtained from climatology. The analytical expressions for the zonal-mean ozone distribution and its gradients agree well with the observed distributions (see Figs. 2b–d), which we evaluate at $15^\circ$N based on a September climatology using data tabulated by McPeters et al. (1984) and Keating and Young (1985). Other seasons yield similar results.

Insertion of (19) and (20) into (14) and (15) yields the following radiative–photochemical feedback terms:

$$a_{\text{OP}} = \frac{ABC}{(B^2 + \omega^2)},$$

(21)

$$a_{\text{VOA}} = \frac{A \omega^2}{N(B^2 + \omega^2)} \left[ \frac{\tilde{\gamma}(y_{\text{EO}},z)}{N} + \frac{a \tilde{\gamma}(y_{15},z) \tilde{D}}{\beta} \right],$$

(22)

$$a_{\text{MOA}} = \frac{AB \tilde{\gamma}(y_{15},z) E^W}{N(B^2 + \omega^2)}.$$

(23)

Appendix C lists, for each wave type, the explicit forms of (21)–(23). If we set $\tilde{\gamma}_{yL} = 0$ in (22), then our (11) reduces to (14) in Cordero et al. (1998), which is valid only for the Kelvin and Rossby–gravity waves.

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2 The form of the vertical mean ozone gradient in (19) differs slightly from Cordero et al. (1998) because we allow the meridional ozone gradient, the first term on the rhs of (18), to vary with height.
b. Limiting cases

We consider several limiting cases in order to clarify the differences and similarities in the wave responses to ozone. The three limits are (i) critical layers \( m_0(\omega, k) \to 0 \), (ii) reflecting layers \( m_0(\omega, k) \to 0 \), and (iii) short waves \( k \to \infty \). Although the WKB approximation may be problematic in limits (i) and (ii), these limits nonetheless provide qualitative guidance in the interpretation of the results that we present later. We list the leading-order ozone physics for each limit in Table 2, where, to ease interpretation, we note that \( a_{OP} \propto C \), \( a_{VOA} \propto \Phi_z \) or \( \Phi_{z\alpha} \), and \( a_{MOA} \propto \Phi_y \).

For all wave types, in the vicinity of a critical layer \( (\omega \to 0) \), \( m_{1,\tau} \) is controlled by \( a_{OP} \) and \( a_{VOA} \), which together cause a contraction in spatial scale; \( m_{1,\delta} \) is controlled by \( a_{NC} \), which causes an increase in spatial damping rate. The controlling physics, however, shows different dependencies on \( \omega \) as \( \omega \to 0 \). For the Rossby–gravity, inertia–gravity, and gravity waves, where \( m_0 \propto \omega^{-2} \), we obtain \( m_{1,\tau} \propto \omega^{-2} \) and \( m_{1,\delta} \propto \omega^{-7/2} \) as \( \omega \to 0 \); for the Kelvin and equatorial Rossby waves, where \( m_0 \propto \omega^{-1} \), we obtain \( m_{1,\tau} \propto \omega^{-1} \) and \( m_{1,\delta} \propto \omega^{-5/2} \) as \( \omega \to 0 \). The fractional dependencies in the exponents for \( m_{1,\delta} \) arise from the Fels (1982) parameterization for Newtonian cooling, where \( a_{NC} \propto \omega^{-1/2} \) as \( \omega \to 0 \).

For reflecting layers, which occur where \( m_0(\omega, k) \to 0 \), the physics-controlling \( m_{1,\tau} \) and \( m_{1,\delta} \) depend on \( \omega \) and wave type (see Table 2). The Kelvin, Rossby–gravity, inertia–gravity, and gravity waves all have reflecting
layers where $\omega \to \infty$ The inertia–gravity and gravity waves have additional reflecting layers where $\omega \to -\infty$; the Rossby–gravity waves have an additional reflecting layer where $\omega \to -\beta/k$, which is also where the equatorial Rossby waves possess their only reflecting layer. For all the wave types, except the Rossby–gravity waves, $m_{1,J}$ and $m_{1,J}$ are controlled solely by $\alpha_{\text{VOA}}$. For the Rossby–gravity waves, $m_{1,J}$ and $m_{1,J}$ are controlled by $\alpha_{\text{VOA}}$ where $\omega \to \infty$, and by $\alpha_{\text{VOA}}$ and $\alpha_{\text{MOA}}$ where $\omega \to -\beta/k$.

In the short wave limit ($k \to \infty$), which is most relevant to the gravity waves, $m_{1,J}$ is controlled by $\alpha_{\text{MOA}}$, while $m_{1,J}$ is controlled by $\alpha_{\text{NC}}$ and $\alpha_{\text{VOA}}$. This latter result shows that for gravity waves, the ozone feedback due to vertical ozone advection is the most important ozone contribution to the spatial damping rate and thus wave driving of the zonal-mean circulation.

4. Results

a. Preliminaries

In this section, we show for each wave type the vertical distributions of the ozone-modified vertical scale $m_{1,J}$, spatial damping rate $m_{1,J}$, and Eliassen–Palm flux divergence $\mathbf{V} \cdot \mathbf{F}$. The distributions are calculated using observed values of zonal-mean ozone, zonal-mean wind, wavenumber, and frequency. For all of our calculations, we use the zonal-mean ozone distribution shown in Fig. 2b and three general zonal-mean wind profiles: the easterly phase of the QBO, the westerly phase of the QBO, and a motionless atmosphere. We discuss our choice of wavenumbers and frequencies next.

Figure 3 shows dispersion curves (thin solid) for the Kelvin, Rossby–gravity, equatorial Rossby, and inertia–gravity waves. The curves in the figure correspond to observed values of zonal-mean ozone, zonal-mean wind, wavenumber, and frequency. For all of our calculations, we use the zonal-mean ozone distribution shown in Fig. 2b and three general zonal-mean wind profiles: the easterly phase of the QBO, the westerly phase of the QBO, and a motionless atmosphere. We discuss our choice of wavenumbers and frequencies next.

In the short wave limit ($k \to \infty$), which is most relevant to the gravity waves, $m_{1,J}$ is controlled by $\alpha_{\text{MOA}}$, while $m_{1,J}$ is controlled by $\alpha_{\text{NC}}$ and $\alpha_{\text{VOA}}$. This latter result shows that for gravity waves, the ozone feedback due to vertical ozone advection is the most important ozone contribution to the spatial damping rate and thus wave driving of the zonal-mean circulation.
wave spectrum contains a wide range of equivalent depths in comparison to the other wave types. We also consider small-scale gravity waves spanning quantized zonal wavenumbers \( 50 < s < 150 \), with corresponding frequencies of \( 3.0 < \sigma < 6.0 \) cycles per day (cpd) (4–8-h periods). These wavenumbers not only fall within the observed gravity wave spectrum (Ern et al. 2004, 2011) but more importantly, they have been shown by Kawatani et al. (2010) to be mostly responsible for driving the zonal-mean circulation (as measured by the EP flux divergence). We do not show the dispersion curves for the gravity waves, however, since the wavenumbers that we consider are outside the wavenumber and frequency range in Fig. 3.

To illustrate the effects of ozone on the waves, we calculate \( m_{1r}, m_{1i} \), and \( \mathbf{V} \cdot \mathbf{F} \) for the waves circled in Fig. 3, and for two waves from the inertia–gravity and gravity wave spectrums (see Table 3).

b. Ozone-modified vertical scale, spatial damping rate, and EP flux divergence

Consider first \( m_{1r}^{-1} \). For our \( \mu = 1 \) ordering of the radiative–photochemical feedbacks (see section 3), changes in vertical scale are solely due to ozone; Newtonian cooling has no effect to order \( \mu \). In the upper atmosphere, irrespective of wave type and zonal-mean wind, ozone causes the vertical scale to contract. In the lower stratosphere, depending on wave type and zonal-mean wind, ozone may cause the vertical scale to expand or contract. To see this, refer to Fig. 4, which shows \( m_{1r}^{-1} \) for each wave type for a motionless atmosphere \( \mathbf{v}(z) = 0 \). For ease of comparison, each profile in Fig. 4 is normalized to its maximum value. Positive (negative) values in Fig. 4 correspond to a contraction (expansion) of the vertical scale. For each wave type, the largest contraction in vertical scale occurs between 35 and 50 km. In this region, \( \alpha_{\text{EP}} \) and \( \alpha_{\text{VOA}} \) make similar contributions to the spatial scale. For a motionless atmosphere, as well as for the wind distributions in Fig. 2a, ozone causes the vertical wave scale to change by about 1%–7% between about 35 and 50 km.

Consider next \( m_{1i} \). Echols and Nathan (1996) and Cordero et al. (1998) have examined the spatial damping rate of Kelvin waves and Rossby–gravity waves for highly select wavenumbers and frequencies. Here we summarize their results, consider a broader range of wavenumbers and frequencies for these waves, and consider the other prominent tropical wave types that they did not consider.

To aid in the interpretation of the effects of ozone on the wave damping, it is instructive to introduce the ratio of the dynamical to photochemical time scales \( \frac{\tau}{\tau_{\text{v}}} \). In the dynamically controlled, transition, and photochemically controlled regions, which extend from the lower stratosphere to the lower mesosphere, respectively, we have \( \frac{\tau}{\tau_{\text{v}}} \approx 1 \), \( \frac{\tau}{\tau_{\text{v}}} \approx 1 \), and \( \frac{\tau}{\tau_{\text{v}}} \approx 1 \).

In the dynamically controlled lower stratosphere \((\sim 20–30 \text{ km})\), where \( \frac{\tau}{\tau_{\text{v}}} \approx 1 \), variations in \( m_{1r}^{-1} \) are mostly due to \( \alpha_{\text{NC}} \) and \( \alpha_{\text{VOA}} \), irrespective of wave type or zonal-mean wind. For \( \frac{\tau}{\tau_{\text{v}}} \approx 1 \), the expression for the spatial damping rate (11) reduces to

\[
m_{1r}^W = -0.5m_0^W \omega^{-1}(\alpha_{\text{NC}} - \alpha_{\text{VOA}}).
\] (24)

Because \( \tilde{\gamma}_z \ll \tilde{\gamma}_z \) throughout the stratosphere, which we have confirmed by numerical calculation, we are able to obtain the simplified expression \( \alpha_{\text{VOA}} \approx A\tilde{\gamma}_z N^{-2} \). Because \( \gamma_z > 0 \) below about 34 km, \( \alpha_{\text{VOA}} \) opposes \( \alpha_{\text{NC}} \) in this region. This means that in the lower
stratosphere, ozone is spatially destabilizing, that is, ozone alone causes the wave to amplify in space. This is consistent with other wave–ozone studies in the tropics and extratropics (Leovy 1966; Zhu and Holton 1986; Nathan and Li 1991; Echols and Nathan 1996; Hitchcock et al. 2010). For the wind distributions shown in Fig. 2a, the vertical ozone advection reduces the spatial damping rate by about 10%–15%.

In the transition region, \( B/\omega \approx 1 \), both ozone advection and ozone photochemistry make significant contributions to the spatial damping rate. In contrast to the dynamically controlled region, the vertical extent of the transition region depends on the wave type and wind distribution. To illustrate this, we show in Fig. 5 the relative contributions of \( \alpha_{\text{NC}}, \alpha_{\text{OP}}, \alpha_{\text{VOA}}, \) and \( \alpha_{\text{MOA}} \) to the spatial damping rate for each wave type. We show the region \( \sim 30\text{–}50 \text{ km} \) and consider first the case where the waves are counter propagating to the zonal-mean wind; we consider the case of waves propagating with the flow later in this section. For westward- (counter) propagating waves, we use the descending westerly phase of the QBO, and for the eastward- (counter) propagating waves, we use the descending easterly phase (see Fig. 2a). In Figs. 5e,f, we show the results for the eastward-propagating inertia–gravity (EIG) and gravity waves (the westward-propagating waves yield essentially the same results). Below 32 km, the spatial damping rate increases as \( \omega \) decreases, consistent with the limiting cases discussed in section 3b.

For the Kelvin wave, the transition region is narrow. Figure 5a shows that \( \alpha_{\text{OP}} \) is the primarily ozone feedback, though \( \alpha_{\text{VOA}} \) also makes a small but significant contribution in the midstratosphere (\( \sim 34\text{–}37 \text{ km} \)). At 47 km \( \alpha_{\text{OP}} \) is maximized and monotonically decreases above and below. For this midstratosphere region, the ozone feedbacks augment Newtonian cooling by about 20%–50%, consistent with previous studies (e.g., Echols and Nathan 1996; Cordero et al. 1998).

For the Rossby–gravity wave, the transition region is broad. Figure 5b shows that all three ozone feedbacks—\( \alpha_{\text{OP}}, \alpha_{\text{VOA}}, \) and \( \alpha_{\text{MOA}} \)—contribute significantly to the spatial damping rate in the transition region. The importance of \( \alpha_{\text{MOA}} \) to the damping of the Rossby–gravity waves makes them unique among all the wave types. Above about 35 km, the three ozone feedbacks together increase the spatial damping rate by about 30%–80%. Below about 35 km, two ozone feedbacks, \( \alpha_{\text{VOA}} \) and \( \alpha_{\text{MOA}} \), predominate and together decrease the spatial damping rate by about 0%–10%. As expected, these results agree with Cordero et al. (1998).

We examine next the equatorial Rossby, inertia–gravity, and gravity waves, which were not considered by either Echols and Nathan (1996) or Cordero et al. (1998). We consider first the equatorial Rossby wave, which, like the Kelvin wave, is characterized by a narrow transition in which \( \alpha_{\text{OP}} > \alpha_{\text{VOA}} \) (see Fig. 5c). Within this narrow region, \( \alpha_{\text{OP}} \) and \( \alpha_{\text{VOA}} \) augment \( \alpha_{\text{NC}} \) to increase the spatial damping rate by about 5%–15% (cf. with the \( \sim 20\%\text{–}50\% \) for the Kelvin wave). For the equatorial Rossby wave, which has a relatively large vertical wavenumber, Newtonian cooling strongly dominates over ozone in the spatial damping rate. This is because the ratio of Newtonian cooling to the ozone heating is \( O(m_{\text{b}}^2) \) as \( m_{\text{b}} \) becomes large.

For the inertia–gravity and gravity waves, the transition region expands with frequency. The effects of ozone on the spatial damping rate in the transition region depends only on \( \alpha_{\text{OP}} \) and \( \alpha_{\text{VOA}} \); \( \alpha_{\text{MOA}} \) is inconsequential. Because \( \bar{\gamma}_g \ll \bar{\gamma}_z \) throughout the stratosphere, the relative importance of vertical ozone advection and ozone photochemistry is measured by the ratio \( \alpha_{\text{VOA}}/\alpha_{\text{OP}} = \omega^2 \bar{\gamma}_g / BCN^2 \). This ratio shows a quadratic dependence on \( \omega \), meaning that near a critical level, photochemistry dominates over advection, and that away from a critical level, advection dominates over photochemistry. Similar to the ratio \( B/\omega \), the ratio \( \alpha_{\text{VOA}}/\alpha_{\text{OP}} = \omega^2 \bar{\gamma}_g / BCN^2 \) is, in slightly disguised form, another way to distinguish the dynamical, transition, and photochemical control regions. Because \( \omega \) increases as we move from the \( n = 0 \) inertia–gravity wave, to the \( n = 1 \) inertia–gravity wave, to the gravity wave, vertical ozone advection becomes increasingly dominant over photochemistry in the spatial damping rate (see Figs. 5d–f). Between about 30 and 50 km, \( \alpha_{\text{VOA}} \) and \( \alpha_{\text{OP}} \) both augment \( \alpha_{\text{NC}} \) to increase the spatial damping rate by about 10%–30%.

In the photochemically controlled region, \( B/\omega \gg 1 \), the expression for the spatial damping rate (11) is approximated by

\[
m_{\text{ij}}^W = -0.5 m_{\text{b}}^W \omega^{-1} (\alpha_{\text{NC}} + \alpha_{\text{OP}}).
\]

Consistent with other studies, ozone photochemistry always augments Newtonian cooling (Leovy 1966; Nathan and Li 1991; Echols and Nathan 1996; Cordero

<table>
<thead>
<tr>
<th>Wave type</th>
<th>( s )</th>
<th>( \sigma ) (cpd)</th>
<th>( c_z ) (m s(^{-1}))</th>
<th>( n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>KW</td>
<td>1</td>
<td>0.1</td>
<td>46.3</td>
<td>-1</td>
</tr>
<tr>
<td>RG</td>
<td>4</td>
<td>-0.25</td>
<td>-29.0</td>
<td>0</td>
</tr>
<tr>
<td>ER</td>
<td>1</td>
<td>-0.1</td>
<td>-46.3</td>
<td>1</td>
</tr>
<tr>
<td>( n = 0 ) IG</td>
<td>5</td>
<td>0.3</td>
<td>27.8</td>
<td>0</td>
</tr>
<tr>
<td>( n = 1 ) IG</td>
<td>12</td>
<td>±0.8</td>
<td>±30.9</td>
<td>1</td>
</tr>
<tr>
<td>GW</td>
<td>80</td>
<td>±5.0</td>
<td>±29.0</td>
<td>80</td>
</tr>
</tbody>
</table>

Table 3. Zonal wavenumber \( s \), ground-based frequency \( \sigma \), zonal phase speed \( c_z \), and meridional wave mode \( n \) for the waves used to calculate \( m_{\text{ij}} \) and \( m_{\text{ij}} \) in Figs. 4 and 5. Because \( |c_z| > |\bar{\gamma}_z| \), none of the waves possess critical levels. Because the \( n = 1 \) and \( n = 2 \) IGs have very similar wave characteristics, we only list below the results for \( n = 1 \). The gravity wave mode \( n = 80 \) has a meridional wavelength that is similar to its zonal wavelength.
et al. 1998; Nathan and Cordero 2007; Nathan et al. 2011; Albers and Nathan 2012). Within the region near 30–65 km, (25) controls the damping rate for waves that propagate with the zonal-mean flow. Recall from section 3b, in the critical layer limit $\omega \to 0$, we have $m_{W0} \sim \omega^{-p}$, where $p = 1$ for the Kelvin and equatorial Rossby waves, and $p = 2$ for the Rossby–gravity, inertia–gravity, and small-scale gravity waves. Using $m_{W0} \sim \omega^{-p}$ in (25), we have for the spatial damping rate $m_{W1,1} \sim \omega^{-(p+1)} (\alpha_{NC} + \alpha_{OP})$ as $\omega \to 0$. From this expression, we see that as the Doppler-shifted frequency $\omega$ decreases, the spatial damping rate increases, and because $\alpha_{NC} \sim \omega^{-1/2}$ as $\omega \to 0$ (see section 3b), $\alpha_{NC}$ becomes increasingly dominant over $\alpha_{OP}$. Thus, compared to waves that propagate counter to the zonal-mean flow (Fig. 5), waves that propagate with the zonal-mean flow are more effectively damped, with the effects of Newtonian cooling tending to dominate over ozone heating. For the wind distributions that we have considered, irrespective of wave type, in the photochemically controlled region, ozone feedbacks augment Newtonian cooling to increase the spatial damping rate by about 10%–20% (not shown).

To calculate the effects of ozone on the wave driving exerted by the waves on the zonal-mean flow, we follow Cordero et al. (1998) and compute the latitudinally integrated EP flux divergence as

Fig. 4. Ozone-modified vertical wavenumber normalized by its maximum value for (a) westward-propagating and (b) eastward-propagating waves in a motionless atmosphere. The wave types include KW, RG, ER, IG ($n = 0$ or $n = 1$), and GW.
\[
\langle \mathbf{V} \cdot \mathbf{F} \rangle = \frac{d\langle F^z \rangle}{dz} = A(z_0) m_{1,j}^W \exp \left( -2 \int_0^z m_{1,j}^W dz' \right),
\]

where \( \langle \cdots \rangle = \int_{-\infty}^{\infty} (\cdots) \, dy \), \( F^z = \rho [(R/H) \beta \gamma N^2 \hat{v}^2 T' - \hat{u}' \hat{w}'] \), and \( A(z_0) \) is the zonal-mean momentum flux at the lower boundary, located at \( z_0 = 16 \) km (~100 hPa). Values for \( A(z_0) \) were obtained from Tindall et al. (2006a).

Because we have chosen the diabatic effects due to Newtonian cooling and ozone heating to be \( O(\mu) \) (see section 3), \( m_{1,j}^W \) is also \( O(\mu) \) and thus does not enter the expression for the latitudinally averaged EP flux divergence (26). The effects of \( m_{1,j}^W \) on the EP flux divergence appear at \( O(\mu^2) \), which we do not consider.

For all of the waves, the EP flux divergence shows a significant response to ozone. As an example, we consider the \( n = 1 \) EIG wave in both westerly and easterly phases of the zonal-mean flow. For the westerly phase, the maximum shear is at 35 km, while for the easterly phase, the maximum shear is at 18 km. These altitudes of maximum shear are consistent with the QBO and yield among the largest responses in \( \langle \mathbf{V} \cdot \mathbf{F} \rangle \) for the EIG wave.
Figure 6a shows, for the EIG wave propagating with the flow in a descending westerly phase of the QBO, the vertical distribution of $\langle V \cdot F \rangle$ for Newtonian cooling alone (solid) and for Newtonian cooling and ozone heating combined (dotted). Ozone causes $\langle V \cdot F \rangle$ to increase by as much as 15% above 33 km and to decrease by as much as 8% below 33 km. For the EIG wave propagating counter to the flow in a descending easterly phase of the QBO, Fig. 6b shows that ozone causes $\langle V \cdot F \rangle$ to increase by as much as 60% above 32 km and to decrease by as much as 12% below 32 km. As expected, the response of $\langle V \cdot F \rangle$ to ozone is consistent with the response of the spatial damping rate to ozone (cf. Figs. 6b and 5e).

5. Conclusions

The tropical stratosphere contains a variety of wave types that span a wide range of spatial and temporal scales. These waves include Kelvin, Rossby–gravity, equatorial Rossby, inertia–gravity, and smaller-scale gravity waves. Here we have derived expressions for the vertical spatial scale and spatial damping rate of these waves that account for feedbacks with ozone. These expressions clearly illuminate how ozone—both its photo-chemistry and its transport—spatially modulates the waves. Moreover, we show that the equations for the spatial damping rate and amplitude combine to yield an expression for the ozone-modified wave driving on the zonal-mean flow. The wave driving provides an efficient parameterization that can be implemented into models that are unable to spontaneously generate ozone-modified, convectively coupled waves that drive the zonal-mean circulation in the tropical stratosphere, including the QBO and SAO. To implement the parameterization only requires the following: background distributions of wind, temperature, and ozone; and, for a given wave type, the zonal wavenumber, meridional mode, frequency, and wave amplitude at the lower boundary. We have built our analysis on an equatorial $\beta$-plane model of the tropical stratosphere, which describes the linear response of a tropical wave field to coupled interactions involving the dynamical circulation, long-wave radiative transfer, ozone transport, and ozone photochemistry. We show that the effects of ozone on the spatial scale, spatial damping rate, and EP flux divergence (wave driving) are strongly modulated by the Doppler-shifted frequency. The ozone effects are maximized in the upper stratosphere, where ozone photochemistry and vertical ozone advection combine to augment Newtonian cooling. This causes a contraction in vertical spatial scale and an increase in spatial damping rate.

We have shown using observed background states of wind, temperature, and ozone that, in the mid- to upper stratosphere, the ozone-induced increase in damping rate and EP flux divergence averages about 10%–30% for all wave types, but that it can be as large as 80% over narrow altitude regions and for specific wave types. In the dynamically controlled lower stratosphere, vertical ozone advection dominates over meridional ozone advection and opposes Newtonian cooling, causing, on average, a 10%–15% reduction in the damping rate and EP flux divergence.

The altitude regions where stratospheric ozone is most effective in modifying the wave, the spatial damping rate
and thus wave driving are precisely the regions where modeling and observational studies show ozone is undergoing changes due to human activities. Chemistry–Climate Models (CCM) show, for example, that for various climate change scenarios, zonal-mean ozone is projected to increase in the tropical upper stratosphere and decrease in the tropical lower stratosphere (Eyring et al. 2007; Butchart et al. 2010). The increase in zonal-mean ozone in the tropical upper stratosphere is attributed to the reduction of ozone-depleting substances (Eyring et al. 2007). The decrease in zonal-mean ozone in the tropical lower stratosphere, however, is attributed to increases in zonal-mean tropical upwelling caused by increases in both CO2 and sea surface temperatures (Lamarque and Solomon 2010). Consistent with the CCM studies, observations show a positive trend in ozone in the upper stratosphere (Randel and Wu 2007). In the lower stratosphere, however, the observations are inconclusive (Jones et al. 2009).

As we discussed in the introduction, modeling studies have made progress in spontaneously generating, without gravity wave parameterizations, convectively coupled waves that drive “QBO-like” oscillations in GCMs. The complexity of these models, however, often poses challenges to understanding the causal relationships between the wave, zonal-mean, and ozone fields. Moreover, using GCMs requires balancing the need for sufficient model resolution to spontaneously generate and accurately reproduce a QBO with the need to reduce computational time in order to obtain reliable statistics, especially in climate studies where multidecadal simulations may be required. For these reasons, the scale-dependent parameterization for the ozone-modified wave driving that we have derived in this study will likely aid both mechanistic and GCM studies of the tropical stratosphere. For mechanistic studies, the parameterizations are computationally efficient and easy to implement. For the GCM studies, the parameterizations can serve as an interpretive tool to understand the role of the ozone-modified wave physics operating in the models.

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APPENDIX A

Dispersion Relations and Spatial Structures at $O(1)$

The dispersion relation for each wave type is written as (Andrews et al. 1987)

\[ m_0^{KW} = -\frac{Nk}{\omega}, \]  
\[ m_0^{RG} = -\text{sgn}(\omega) \frac{N}{\omega^2} (\beta + \omega k), \]  
\[ m_0^{ER} = \frac{N\beta}{\omega^2} \left\{ \left( n + \frac{1}{2} \right) + \frac{\omega k}{\beta} \left( 1 + \frac{\omega k}{\beta} \right) \right\}^{1/2}, \]  
\[ m_0^{IG,GW} = -\text{sgn}(\omega) \frac{N\beta}{\omega^2} \left\{ \left( n + \frac{1}{2} \right) + \frac{\omega k}{\beta} \left( 1 + \frac{\omega k}{\beta} \right) \right\}^{1/2}, \]

where the superscripts denote wave type: $KW =$ Kelvin wave, $RG =$ Rossby–gravity, $ER =$ equatorial Rossby, $IG =$ inertia–gravity, and $GW =$ small-scale gravity wave. The dispersion relation for the Kelvin wave is a special case, corresponding to $n = -1$ in (9).

The lowest-order spatial structures of the wave fields are

\[ \begin{pmatrix} u', v', w', T', \Phi', \gamma' \end{pmatrix}^{KW} = S^{KW} \begin{pmatrix} 1, 0, \frac{\omega}{N}, \frac{HN}{R}, \frac{\omega}{k}, \frac{1}{N} \gamma_z \end{pmatrix}, \]  
\[ \begin{pmatrix} u', v', w', T', \Phi', \gamma' \end{pmatrix}^{RG} = S^{RG} \begin{pmatrix} \frac{i|m_0^{RG}| \omega y}{N}, 1, -\frac{i|m_0^{RG}| \omega^2 y}{N^2}, \frac{|m_0^{RG}| H \omega y}{R}, i \omega y, \frac{i}{\omega} \left( \gamma_y - \frac{i|m_0^{RG}| \omega^2 y}{N^2} \gamma_z \right) \end{pmatrix}, \]
\[
S^W = \Re^W \exp\left(-\frac{\eta^2}{2}\right). \tag{A8}
\]

In (A7) the \(H_n\) represent Hermite polynomials; \(H_0 = 1, H_1 = 2\eta, H_3 = 4\eta^2 - 2\), etc., where \(\eta(x,y) = (\beta|m_0^W|N^{-1})^{1/2} y\).

\[
H_+^W = \left[\frac{1}{2} \frac{H_{n+1}(\eta)}{b_+} - \frac{nH_{n-1}(\eta)}{b_+}\right], \tag{A9a}
\]

\[
H_-^W = \left[\frac{1}{2} \frac{H_{n+1}(\eta)}{b_-} + \frac{nH_{n-1}(\eta)}{b_-}\right], \tag{A9b}
\]

\[
b_- = |m_0^W| \omega - Nk, \quad \text{and} \tag{A10a}
\]

\[
b_+ = |m_0^W| \omega + Nk. \tag{A10b}
\]

**APPENDIX B**

**Wave Amplitudes**

The slowly varying amplitude \(\Re\) for all wave types is represented as

\[
\Re^W(y,\zeta) = a_0|m_0^W|^{1/4} \exp\{\Pi(y,\zeta;m_0^W)\}. \tag{B1}
\]

For the Rossby–gravity, equatorial Rossby, inertia–gravity, and gravity waves

\[
\Pi = \Pi_1 + \Pi_2 + \Pi_3, \tag{B2}
\]

where

\[
\Pi_1 = \ln|m_0^W|^{n/2}, \tag{B3}
\]

\[
\Pi_2 = -\int_{\zeta_0}^{\infty} c_1 J^{-1} \left[\int_{-\infty}^{\infty} \left(\tilde{H}_-^W \tilde{H}_+^W\right)_\zeta dy\right] d\zeta, \tag{B4}
\]

\[
\Pi_3 = \int_{\zeta_0}^{\infty} \left(\frac{\ln|m_0^W|}{2|m_0^W|} J^2(b_+^2 - n(n+1)b_+ b_- - n b_-^2)\right) d\zeta, \tag{B5}
\]

In (B3)–(B5), \(\tilde{H}_-^W, b_-,\) and \(b_+\) are defined in appendix A. In (B4) \(c_1\) and \(J\) are defined as

\[
c_1 = \frac{1}{2^{n+1} n!} \sqrt{\frac{\beta|m_0^W|}{\pi N}}, \tag{B6}
\]

\[
J = \frac{b_+^2 (n+1) + b_-^2 n}{b_+^2 b_-^2}. \tag{B7}
\]

For the Kelvin wave, \(\Pi\) is defined as

\[
\Pi = -\ln|\omega|^{1/4}. \tag{B8}
\]

The functions \(D\) and \(E\) that appear in the radiative–photochemical terms \(\alpha_{\text{VOA}}\) and \(\alpha_{\text{MOA}}\) in (14) and (15) arise from the inner products \(\{w'T^W\}^*\) and \(\{v'T^W\}^*\), which for the Rossby–gravity, equatorial Rossby, inertia–gravity, and gravity waves are defined as

\[
D(y;m_0^W) = c_1 J^{-1|m_0^W|^{1/2} \left[\frac{iA\omega}{N^2(B-i\omega)}\right] (H_+^W H_-^W) \exp(-\eta^2), \tag{B9}
\]

\[
E(y;m_0^W) = -c_1 J^{-1|m_0^W|^{1/2} \left[\frac{BN|m_0^W|}{2\omega}\right] (H_+^W H_-^W) \exp(-\eta^2), \tag{B10}
\]

where \(c_1, J^{-1}, H_+, H_-\), and \(\eta\) are defined in appendix A.
The functions $D$ and $E$ for the Kelvin wave are defined as

$$D(y; m_0^{KW}) = c_2 \frac{m_0^{KW}}{2\omega} \left[ \frac{i\omega}{N^2(B - i\omega)} \right] \exp(-\eta^2), \quad (B11)$$

$$E(y; m_0^{KW}) = 0, \quad (B12)$$

where

$$c_2 = \sqrt{\frac{\beta|m_0^{KW}|}{\pi N}}. \quad (B13)$$

**APPENDIX C**

**Ozone Feedback Terms for Each Wave Type**

The analytical expressions of the zonal-mean ozone gradients in (22) and (23) yield the following expressions for the ozone feedback terms:

$$\{\alpha_{OP}, \alpha_{VOA}, \alpha_{MOA}\}^{KW} = \frac{A}{B^2 + \omega^2} \left[ BC, \frac{\omega^2}{N} \left( \frac{\nabla y}{N} + \frac{\omega a\nabla y}{2B\eta N} \right), 0 \right], \quad (C1)$$

$$\{\alpha_{OP}, \alpha_{VOA}, \alpha_{MOA}\}^{ER} = \frac{A}{B^2 + \omega^2} \left[ BC, \frac{\omega^2}{N} \left( \frac{\nabla y}{N} + \frac{\omega a\nabla y}{2B\eta N} \right), \frac{Ba\nabla y}{N(\beta + \omega k)} \right], \quad (C2)$$

$$\{\alpha_{OP}, \alpha_{VOA}, \alpha_{MOA}\}^{IG,GW} = \frac{A}{B^2 + \omega^2} \left[ BC, \frac{\omega^2}{N} \left( \frac{\nabla y}{N} + \frac{\omega a\nabla y}{2B\eta N} \right), \frac{Ba\nabla y}{N \beta} \right], \quad (C3)$$

$$\{\alpha_{OP}, \alpha_{VOA}, \alpha_{MOA}\}^{W} = \frac{1}{2|m_0^{W}|} \left[ (n+1)(n+\frac{3}{2})b_+^2 - n(n+1)b_-b_+ + n(n-\frac{1}{2})b_-^2 \right]. \quad (C5)$$

$$E^{W} = \frac{1}{m_0^{W}} \left( \frac{n+1}{n+1}b_-b_+^2 - nb_-^2 b_+ \right). \quad (C6)$$

The $b_-$ and $b_+$ are defined in appendix A.

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